Uncertainty Quantification of Soil-Structure Interaction in Tunnel Linings by Polynomial Chaos Expansion

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Abstract. The paper is focused on uncertainty quantification of soil-structure interaction in tunnel linings using a surrogate model in form of Polynomial Chaos Expansion (PCE). Tunnel design is a complex and complicated task since it is strongly associated with a great number of load and material uncertainties. Moreover, modelling the soil-structure interaction multiplies the complexity and non-linearity of a tunnel engineering problems. In order to handle such uncertainties, finite element method with random input variables has proven to be a very accurate tool. The probabilistic analysis is typically performed by Monte Carlo simulation (MC), simulating uncertainties according to their complete probability distributions and statistical correlations. The computational burden of MC represents the main obstacle to its use in complex numerical models and it is therefore not practical for industrial applications. The solution can be an efficient approximation of the original mathematical model by computationally efficient analytical function – a surrogate model. In this study, the surrogate model in form of PCE is utilized, allowing for analytical post-processing (statistical and sensitivity analysis). Uncertainty quantification is focused on estimation of spatial variability of internal forces caused by the soil-structure interaction.

Keywords: Uncertainty Quantification · Polynomial Chaos Expansion · Spatial Variability · Tunnel Linings · Concrete Structures

1 Introduction

With the rapid advancement of a computer technology and the growing demand for cost-effective and complex design solutions in the construction industry, numerical modeling has emerged as the preferred approach in various structural and heavy civil engineering domains, including tunnel engineering. Finite element (FE) methods have become essential tools, proving their efficacy in analyzing complex underground structures by advanced constitutive materials and detailed geometrical simulations. Additionally, numerical modeling facilitates the analysis of intricate geometries, encompassing non-circular cross-sections, unique interfaces, and junctions.

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In tunnel design and assessment, uncertainties arising from knowledge gaps, randomness in soil/rock performance, structure properties and their interactions, or mathematical indeterminism necessitate the consideration of safety formats and risk-based decisions. The following four analysis methodologies in literature and design standards quantify these uncertainties in terms of failure probabilities or structural reliability measures:

- Level I: Uncertain parameters are modelled by a single nominal value, e.g. characteristic values with partial safety factor design.
- Level II: Uncertain parameters are implicitly modelled with a probability distribution described by two characteristics, e.g. mean and variance or characteristic value, i.e.. Semi-probabilistic methods.
- Level III: Uncertain quantities are modelled by their distribution functions, and correlations, leading to a calculated failure probability, i.e. full probabilistic Monte Carlo simulations.
- Risk-based methods: The consequences of failure are also accounted as a design criterion.

Despite the prevalence of Level I methods amongst practitioners, more advanced approaches are gaining favor due to their efficiency and rationalization in engineering verifications. Nevertheless, risk-based and probabilistic methods face challenges in their implementation due to their reliance on multidisciplinary expertise and computational power. “Level II” methods are advantageous since they offer a balance between an accuracy and an efficiency as was shown in the previous work of the authors [1].

This study continues in exploration of the possibilities of advanced analysis methods in tunnelling, particularly it is focused on the Level III method. The full probabilistic approach of costly mathematical models is unfortunately not feasible in practical applications and thus we investigate possibilities of surrogate modeling for uncertainty quantification instead of crude Monte Carlo simulation. The paper initially introduces theoretical background of polynomial chaos expansion and formal model descriptions. Comparative calculations are then demonstrated using a plane-strain FE model from a realistic tunnel project in soft soil and obtained results based on surrogate modelling are compared to a MC with original FE model as a reference solution.

2 Polynomial Chaos Expansion

Considering a model response $Y = f(\mathbf{X})$ to be a random variable referenced as a quantity of interest (QoI) with finite variance σ^2, the polynomial chaos expansion of QoI is in the following form [2]:

$$Y = f(\mathbf{X}) = \sum\nolimits_{\alpha \in N^M} \beta_\alpha \psi_\alpha(\boldsymbol{\xi}), \tag{1}$$

where M is the number of input random variables, $\beta_{\boldsymbol{\alpha}}$ are unknown deterministic coefficients and $\psi_{\boldsymbol{\alpha}}$ are multivariate basis functions orthonormal with respect to the joint probability density function (PDF) of $\boldsymbol{\xi}$. The basis functions must be selected in dependence to probability distributions of components of the input random vector $\mathbf{X}$ which must be transformed to the associated standardized variables ξ [3].

Further, it is necessary to truncate PCE to a finite number of terms P. Truncated set of basis functions $\mathcal{A}^{\mathcal{M},p}$is typically dependent on given maximal polynomial order p and M as follows:

$$\mathcal{A}^{\mathcal{M},p} = \{\alpha \in N^M : |\alpha| = \sum_{i=1}^{M} \alpha_i \ \le p\} \tag{2}$$

Deterministic coefficients $\beta_{\boldsymbol{\alpha}}$ can be obtained by intrusive and non-intrusive approaches. Non-intrusive methods utilize the original mathematical model (e.g. FE) as a black-box, which allows for their easy applications in combination with commercial software and thus is typically preferred for industrial applications. The most popular type of the non-intrusive approach is based on a simple linear regression.

The original mathematical model must be evaluated to obtain a vector of results corresponding to generated sample points. Once the basis functions are created and experimental design (ED) is prepared, PCE coefficients can be estimated by ordinary least square (OLS) regression method. Unfortunately, a truncated PCE solved by OLS is not highly computationally efficient and cannot be employed for practical examples with large number of input random variables due to the *curse of dimensionality*. The solution is an additional reduction of the truncated basis set by any model-selection algorithm such as Least Angle Regression (LAR) [4]. LAR automatically detects the most important basis functions for given ED and create the so called sparse PCE. For further reduction of computational cost, it is beneficial to employ advanced sampling schemes for a sequential enrichment of ED [5]. In this paper, UQPy implementation [6] of an algorithm for construction of a non-intrusive sparse PCE based on LAR is employed for numerical examples. Powerful post-processing is main advantage of PCE over to other surrogate models. First of all, thanks to the explicit form of PCE it is possible to obtain a leave-one-out error Q^2 analytically without additional computational demands which can be further used for an adaptive construction of PCE approximation. Besides the analytical derivation of Q^2, it is possible to derive also the first four statistical moments directly from deterministic coefficients [7]. Estimated statistical moments can be further utilized for a global sensitivity analysis [8].

3 Numerical Model

3.1 Model Set-Up

A 2D plane-strain FE model (see Fig. 1) has been developed and analyzed using Abaqus FE code, featuring the excavation of a sprayed concrete lined tunnel with a cross-sectional area of 50 m^2 in a soil medium at a depth of 20 m to the tunnel axis. The tunnel is about 7 m high and 9.5 m wide and consists of 4 radii.

The analyses included three steps:

– a geostatic step where the initial soil pressures were estimated from the density of the soil material and the specified coefficient of lateral earth pressure,

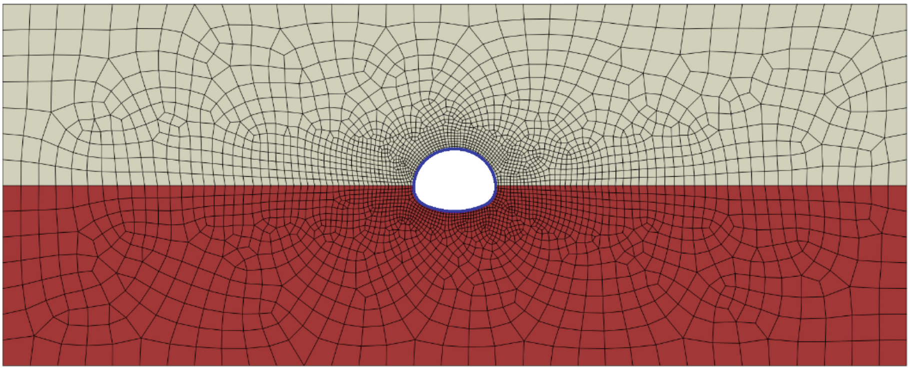

Fig. 1. Mesh of the finite element model.

- a relaxation step simulating the stress relief and 3D arching effects during excavation. This has been realized through the stiffness reduction method, where the soil within the excavation was assigned lower parameters (Young's modulus, Poisson's ratio) allowing the soil to deform to a new equilibrium before the lining installation (refer to details of this modelling approach and methodology in [9, 10],
- the undrained excavation step, where the elements inside the excavation were removed in a single step and the tunnel support was activated.

The FE mesh included 3740 four-node plane-strain elements for the soil (type CPE4R) and 86 beam elements (type B21) for the lining. The soil elements were assigned a non-linear Morh-Coulomb plasticity. The sprayed concrete lining elements were modelled with a linear-elastic material model. To account for the stiffer soil response during unloading and reloading stress paths, the soil elements below the tunnel axis were assigned a three times higher stiffness.

3.2 Random Input Parameters

The geological model and the range of parameters used have been selected based on an extensive survey of available geotechnical information in [11, 12] and they are anticipated to be representative of a typical shallow tunneling project in urban environment. In summary we assumed the following uncertain parameters: Young's modulus of concrete E_c [13] coefficient of lateral earth pressure K_0, undrained shear strength S_u, ratio between the Young's modulus of the soil and S_u and relaxation factor κ, while all other parameters are kept as deterministic. An overview of the uncertain parameters used is given in Table 1. The concrete liner was modelled with a constant thickness of 300 mm and a Poisson's ratio of $\nu = 0.2$.

Table 1. Input stochastic model parameters, indicating the mean values, the standard deviations and (in parenthesis) the coefficients of variation for Normal and Lognormal variables or upper and lower bounds for Uniform distributions.

Input Variable	Mean [units]	Std (CoV) [bounds]	Distribution
Young's modulus E_c	13 [GPa]	1 (0.08)	Lognormal
E_s/S_u ratio	1000	150 (0.15)	Normal
Undrained shear strength s_u	250 [kPa]	50 (0.2)	Normal
Lateral stress coefficient K_o	1 [–]	[0.4, 1.6]	Uniform
Relaxation factor κ	0.5 [–]	[0.1, 0.9]	Uniform

3.3 Methodology

The FE model of the concrete lining consists of 86 finite elements and three QoIs in each element (internal forces N, V and bending moments M) are approximated by separate individual PCEs, i.e. totally 86×3 surrogate models were constructed and further utilized for UQ of the whole structure. Each PCE is constructed by the adaptive sparse algorithm implemented in UQPy Python package and it is based on ED containing 50 samples generated by Crude Monte Carlo. The maximum polynomial order is automatically selected as $p \in [3, 10]$ and a sparse basis set is identified by LAR. Polynomial basis consists of Legendre and Hermite polynomials associated to Uniform and Normal random variables respectively, input Young's modulus is thus first transformed to a Gaussian space by an iso-probabilistic transformation. Note that a computational cost of construction of a single PCE is less than a second and thus it is possible to construct surrogate models for all finite elements in a few minutes and further use them for UQ without any additional computational cost. In this study, the main task is a statistical analysis – an estimation of local mean values $\mathbb{E}[\mathrm{Y}|x]$and standard deviations $\sigma[\mathrm{Y}|x]$ of internal forces (normal and shear) and bending moments. Design and assessment of structures is typically based on extreme quantiles estimated from mean values, variances, and prescribed reliability indices dependent on specific types of structures [14]. Therefore, for the sake of generality, we estimate also quantiles corresponding to $\pm 3\sigma$ in the example.

4 Numerical Results and Discussion

The numerical results are depicted in Fig. 2. The statistical moments and quantiles of internal forces obtained from PCE (right column) are compared to a reference solution obtained by MC with 1000 simulations (middle column) and also an estimation from ED generated by MC and utilized for a construction of PCE containing 50 samples (left column). As can be seen from identical shapes of internal forces shown in Fig. 2, PCE leads to consistent results. Detailed insight to numerical results can be seen in Fig. 3. It is clear that although a direct estimation from 50 samples leads to a significant under-estimation of mean values and variance (clearly visible in Fig. 3 for normal forces), PCE based on the identical samples, and thus identical computational cost, significantly improves the accuracy of an estimation. This is caused by the fact, that PCE considers the whole probability distributions of input variables through basis functions instead of limited point-wise information from samples. Errors can be clearly seen in Fig. 3 showing relative absolute errors between estimations and the reference solution. The most significant improvement can be seen in both local mean values and standard deviations of normal forces, while the least significant improvement is for shear forces. However, extreme values of relative errors are associated to locations with shear forces close to zero, and thus such errors are not visible in Fig. 2.

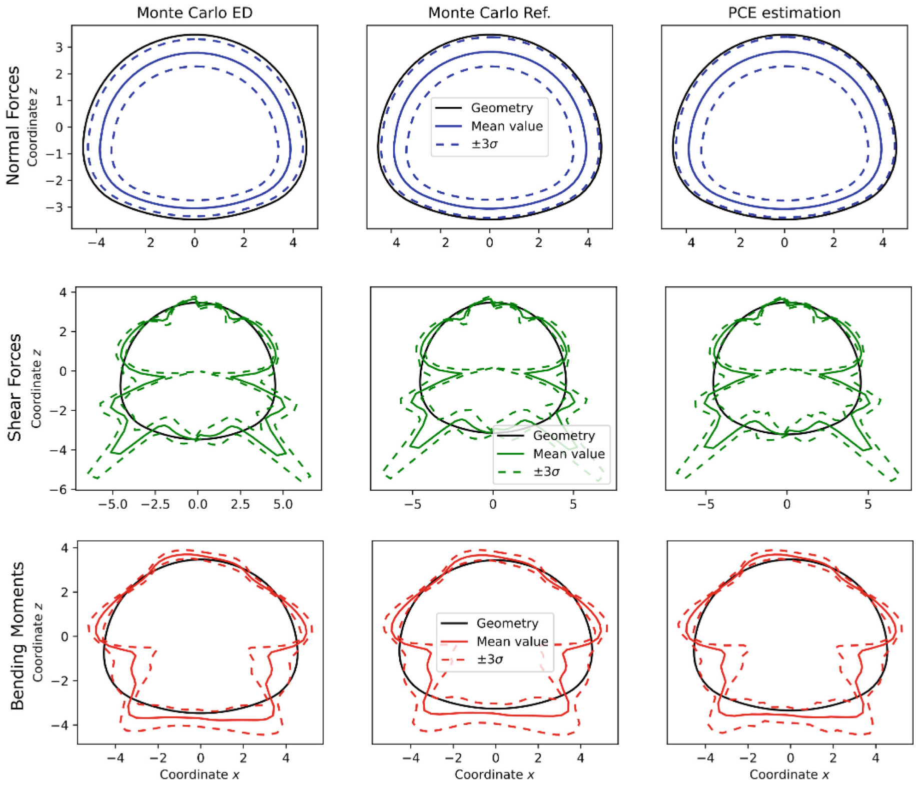

Fig. 2. Geometry of the concrete liner (black) together with estimations of local mean values (solid) and extreme quantiles (dashed) of normal forces (blue), shear forces (green) and bending moments (red) by PCE (right), Monte Carlo with 1000 samples (middle) and 50 samples (left).

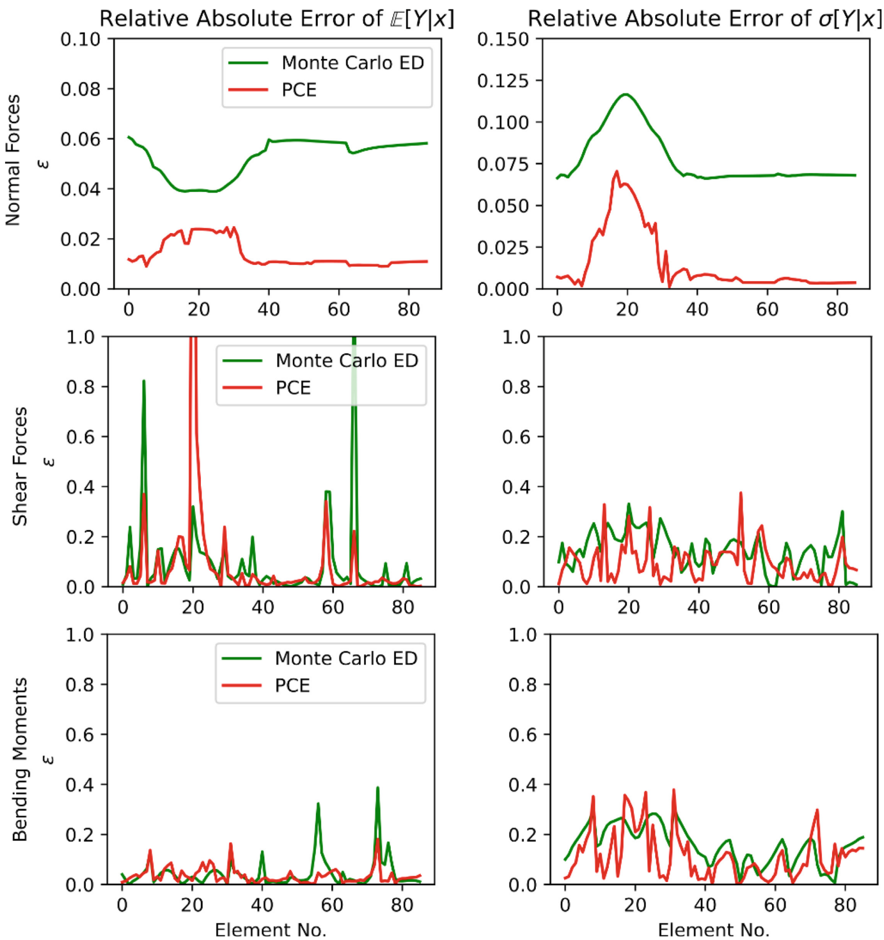

Fig. 3. Relative absolute error of local mean values (right) and standard deviations (left) of normal forces (top), shear forces (middle) and bending moments (bottom) for each element number (both methods are based on 50 samples in ED)

5 Conclusions

The paper explores possibilities of PCE for UQ as a LEVEL III method for design and assessment of tunnel linings. The main task was an estimation of uncertain internal forces caused by a soil-structure interaction affected by input random parameters. It was shown that PCE is able to greatly reduce the number of numerical simulations in a comparison to a standard crude MC approach while achieving satisfactory accuracy of estimated statistical moments. Moreover, thanks to a negligible computational cost of adaptive sparse solvers for construction of PCE, it was possible to create an independent PCE for each finite element of the tunnel liner. The obtained set of PCEs was further utilized for an analytical post-processing – an estimation of local mean values and standard deviations. Naturally, besides accurate UQ of QoIs, PCE can be also utilized further as a standard surrogate model for various additional simulations (e.g., a reliability analysis), and thus it offers an additional value in comparison to standard Monte Carlo analysis though it might lead to comparable estimation of statistical moments.

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